crossings of this line with the lines l_3, l_4 and l_5 are S_3, S_4 and S_5. These points are centers of the circles C_3, C_4 and C_5 passing through the points F_∞ and A.

To obtain a measure for the reactance of the external circuit, we define

$$Q_1 = \frac{\omega L_1}{R_1 + R_K}, \tag{4}$$

the value of which is determined by the radii r_2 and r_5 as follows:

$$Q_1 = \frac{r_2}{r_5}. \tag{5}$$

For small values of Q_1 the point S_5 falls far outside the Smith Chart, and it is therefore more convenient to determine Q_1 from the angle α.

$$Q_1 = \operatorname{tg} \alpha. \tag{6}$$

The crossings of the circles C_3, C_4 and C_5 with the Q circle C_l are the points F_3, F_4 and F_5. The corresponding frequencies f_3, f_4 and f_5 are found on the frequency scale. The Q value of the unloaded resonator is then given by

$$Q_0 = \frac{f_5}{f_4 - f_2}. \tag{7}$$

The frequency f_5 is the resonant frequency of the unloaded resonator. Of course, Q_0 may be evaluated also from

$$Q_0 = (1 + \kappa) Q_L. \tag{8}$$

As it can be seen, the described method permits the evaluation of resonator parameters by a straightforward graphical analysis on the Smith Chart. There is, therefore, no need for auxiliary digrams, which are necessary in the other existing methods[2-4]

The limitation of the described method consists in the fact that it is usable only in cases where the losses in the coupling circuit can be represented by a series resistance only. It is possible, however, to modify the method also for such cases where the losses are represented by a parallel resistance only. The method becomes rather complicated if the losses are to be represented by a combination of series and parallel resistance.[5]

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Calculating Coaxial Transmission-Line Step Capacitance*

As a frequent user of the curves given by J. R. Whinnery, H. W. Jamieson and T. E. Robbins,[1] I found it useful to arrive at a simple polynomial in powers of α and τ, which makes it possible to incorporate the step-capacitance calculation in a subroutine in a computer program dealing with transmission line calculations in coaxial lines.

* Received June 5, 1963.

[1] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial line discontinuities," PROC. IRE, vol. 32, pp. 695–709; November, 1944.

TABLE I

	Step on Inner		
	a_i	b_i	c_i
$i=1$	−0.771	+12 6	+ 58.2
2	+1.49	−23.8	−100.0
3	−0.778	+13.1	+ 27.7
4	+0.041 9	− 1.53	+ 15 5
5	+0.000 92	+ 0.069 9	− 0.726

	Step on Outer		
	a_i	b_i	c_i
$i=1$	−0 606	−4.100	+ 82 0
2	+1.13	+3.63	−138.6
3	−0 482	−1.36	+ 48.6
4	−0.115	+1.92	+ 10.2
5	+0.024 0	−0.182	− 0.397

From considering the peculiarities of the curves (Fig. 8, and Fig. 9 in the above mentioned article), the following form was chosen:

$$\begin{aligned} c'_d = {} & (a_1\tau^2 + b_1\tau + c_1)\alpha^2 \\ & + (a_2\tau^2 + b_2\tau + c_2)\alpha^1 + \cdots \\ & + (a_5\tau^2 + b_5\tau + c_5)\alpha^{-2} \text{ mpf/cm.} \end{aligned}$$

The coefficients for both cases (step on inner, step on outer) are given in Table I. These coefficients give a perfect fit at points $\alpha = 0.1$, 0.3, 0.5, 0.7, 0.9, and $\tau = 1$, 3, 5, and yield an accuracy of "line thickness" at any other point between the limits $0.1 \leq \alpha \leq 1.0$, $1 \leq \tau \leq 5$.

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Field Measurements Using Active Scatterers*

A general theory for analyzing scattering from loaded scatterers is available, and has been applied to small scatterers suitable for electromagnetic field measurements.[1] The theory is valid for both passive and active loads, as long as the load is linear. Ryerson has proposed the use of tunnel diodes to provide a negative resistance load, thereby enhancing the scattered signal.[2] His predictions have been verified experimentally by measurements on dipoles and tunnel diodes at S band.[3] The use of scatterers and tunnel diodes for field measurements is discussed in this communication.

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[1] R. F. Harrington, "Small resonant scatterers and their use for field measurements," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 165–174, May, 1962.

[2] J. L. Ryerson, "Scatter echo area enhancement," PROC. IRE (*Correspondence*), vol. 50, p. 1979, September, 1962.

[3] J. Forgione, E. Calucci, and C. Blank, "The Application of Tunnel Diodes to a Reflecting Antenna Array," Applied Research Lab., Rome Air Development Center, Griffiss Air Force Base, N. Y., T.D. Rept. No. RADC-TDR-63-4; January, 1963.

The primary purpose of loading a small scatterer is to increase its echo area. The general formula is given by (18) of Harrington,[1] but in most cases the second term is large compared to the first term. Also, by reciprocity, the B of (18) is the scatterer gain times its input resistance; hence

$$\frac{\sigma}{\lambda^2} \approx \frac{1}{\pi} \left| \frac{G R_{in}}{Z_{in} + Z_L} \right|^2, \tag{1}$$

where σ = echo area, λ = wavelength, $Z_{in} = R_{in} + jX_{in}$ = the input impedance of the scatterer when used as an antenna, G is the directive gain of the scatterer when used as an antenna, and Z_L is the load impedance connected to the scatterer terminals. Note that the echo area is completely determined by the characteristics of the scatterer when used as an antenna. The extension of (1) to the case of bistatic scattering involves merely the replacement of G^2 by G_1G_2, where G_1 is the gain in the direction of the source and G_2 is the gain in the direction of the receiver.

By using a negative resistance load, one can make the denominator of (1) arbitrarily small, obtaining very large echo areas from small scatterers. In practice, the maximum echo area is limited by instabilities that arise. Some of the characteristics of small scatterers with negative resistance loads that are of importance in field measuring techniques are as follows. 1) The scatterer becomes extremely sensitive to proximity effects, because a small change in Z_{in} results in a large change in σ. Hence, such scatterers might be useful for the measurements in regions distant from objects, but probably not close to objects. 2) The scatterer behaves similarly to a resonant circuit with an effective quality factor

$$Q = \frac{|X_{in}|}{R_{in} + R_L}, \tag{2}$$

which becomes very large when R_L is negative. Hence, the scatterer becomes a very narrow-band device. This may be an advantage if a frequency-modulated system is used, as discussed in Sec. VIII of Harrington[1] 3) Because the scatterer behaves as a resonant circuit, it can be shown that a scatterer is characterized by a constant gain-bandwidth product, that is,

$$\sigma\beta^2 = \text{constant}, \tag{3}$$

where β = fractional frequency bandwidth between points where σ has fallen to 1/2 its value at resonance. Fig. 1 illustrates this